

**MATH 521**  
**Methods of Applied Mathematics**  
Fall Semester 2010

**Professor: Vianey Villamizar**

**Class: 3:00 - 3:50 p.m. MWF B030 JFSB**

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**Office Hours: Problem Session Tuesday 5:00-6:30 pm (at XXXX), and Fr 4:00-5:00 pm (office).**

**Text: Applied Mathematics (Third edition) by David Logan. Wiley-Interscience, 2006 [DL].**

**OUTLINE**

**1. Fluid Flow Past a Circular Cylinder (Notes)**

- 1.1 Navier-Stokes Equations.
- 1.2 Dimensional Analysis and Boundary Layer Equations.
- 1.3 Review of Vector Calculus.
- 1.4 Potential Flow Theory.
- 1.5 Flow Past a Circular Cylinder.

**2. Partial Differential Equation (PDE) Models. [6.2 - 6.3 DL][H]**

- 2.1 Diffusion Equations
  - 2.1.1 The Heat Equation.
  - 2.1.2 Well-Posedness
- 2.2 Equilibrium Equations
  - 2.2.1 Laplace's Equation.
  - 2.2.2 Integral Identities.
  - 2.2.3 Well-Posedness.

**3. Green's Functions [Chap 9 H][4.4 DL]**

- 6.1 Green's Functions of BVP for Ordinary Differential Equations.
- 6.2 The Dirac Delta Function.
- 6.3 Fredholm Alternative.
- 6.4 Green's Function for Poisson's Equation. Method of Images.

**3. Infinite Domain Problems:**

**Fourier Transform Solutions of PDE. [4.1, 6.5 DL][Chap 10 H]**

- 3.1 Complex Form of Fourier Series.
- 3.2 Discrete Finite Fourier Transform.
- 3.3 Fast Fourier Transform. Numerical Examples.
- 3.4 Fourier Transform Pair.
- 3.5 Application to the Heat Equation.
- 3.6 Fourier Sine and Cosine Transform.
- 3.7 One and Two-Dimensional Examples.

#### 4. Stability of Solutions and Distributions [6.6 – 6.7 DL]

#### 5. Integral Equations [4.3 DL]

- 4.1 Volterra Equations.
- 4.2 Fredholm Equations with Separable Kernels. Alternative Theorems
- 4.3 Symmetric Kernels. Hilbert-Schmidt Theorem.
- 4.4 Numerical Solution of Fredholm Integral Equations.

#### 6. Nonlinear Waves. Traffic Flow [12.6 H][7.1 – 7.3 DL]

- 5.1 Method of Characteristics.
- 5.2 Shock and Expansion Waves.
- 5.3 Entropy Condition.
- 5.4 Application to Traffic Flow.
- 5.5 Burger's and KdV Equations.

**Either Section 7, 8, or both if time permits.**

#### 7.- Dimensional Analysis and Asymptotic Methods [DL]

- 7.1 Buckingham Pi Theorem. Scaling
- 7.2 Regular Perturbation.
- 7.3 Singular Perturbation.
- 7.4 Back to our Real World Problem. Boundary Layer Analysis

#### 8.- Field Equations of Continuum Mechanics [DL]

- 8.1 Kinematics.
- 8.2 Conservation Laws.
  - 8.2.1 Conservation of Mass.
  - 8.2.2 Balance of Linear Momentum.
  - 8.2.3 Balance of Angular Momentum
  - 8.2.4 Energy and Entropy
- 8.3 The Navier-Stokes Equations.

### REFERENCE LIST

- [LS] C. C. Lin and L. A. Segel, *Mathematics Applied to Deterministic Problems in the Natural Sciences*, SIAM, 1988.
- [DF] D. G. Duffy, *Green's Functions with Applications*, Chapman & Hall/CRC, 2001.
- [OHL] J. Ockendon, S. Howinson, and A. Lacey, *Applied Partial Differential Equations*, Oxford University Press, 2003.
- [S] L. E. Segel, *Mathematics Applied to Continuum Mechanics*, Macmillan Publishing Co., Inc, 1977.
- [H] R. Haberman, *Applied Partial Differential Equations*, 4<sup>th</sup> Edition, Pearson Prentice Hall, 2004.
- [G] M. S. Gockenbach, *Partial Differential Equations* SIAM, 2002.
- [DR] L. Dresner, *Applications of Lie's Theory of Ordinary and Partial Differential Equations*, IOP Publishing Ltd, 1999.
- [TS] A. N. Tikhonov and A. A. Samarskii, *Equations of Mathematical Physics*, Macmillian, 1963.

[DF2] D. G. Duffy, *Transform Methods for Solving Partial Differential Equations*, Chapman & Hall/CRC, 2004.

### **Homework**

Homework will be assigned every week with their corresponding due date. A discussion of the solutions of homework problems will be held during a **weekly review session on Thursday evenings**. This is optional but I can anticipate that it will be of great benefit for the students. Discussion of homework assignments is recommended, but you should keep in mind that homework is an individual work. If you can reach to the point where you can do fresh problems without help in all sections, I can anticipate that you will be able to successfully solve all problems of the midterms and final exam. Students will assist in the homework grading.

### **Course grade**

Course grade will consist of

Homework	20 %	
First Midterm Exam	25%	October 7-9 Testing Center (3 hours approx.)
Second Midterm Exam	25%	November 11-13 Testing Center (3 hours approx)
Final Exam	30%	Friday December 17 7:00- 10:00 a.m. B030 JFSB

At the end of the semester, I will make an average of each one of the above forms of evaluations with their corresponding weights. Then a Gaussian curve will help me to determine your final grade. Keep in mind that a good grade is the end result of a good learning process. All of you can get a good grade by successfully experiencing this learning process.

**Preventing Sexual harassment:** BYU's policy against sexual harassment extends not only to employees of the university but to students as well. If you encounter sexual harassment, gender-based discrimination, or other inappropriate behavior, please talk to your professor or department chair, or contact the BYU Equal Employment Opportunity Office at 422-5895, or contact the Honor Code Office at 422-2847.

**Students with disabilities:** BYU is committed to providing reasonable accommodation to qualified persons with disabilities. If you have any disability that may adversely affect your success in this course, please contact the University Accessibility Center at 422-2767. Services deemed appropriate will be coordinated with the student and instructor by that office